Chapter 35 Solutons

35.1 The Moon's radius is 1.74×10^6 m and the Earth's radius is 6.37×10^6 m. The total distance traveled by the light is:

$$d = 2(3.84 \times 10^8 \text{ m} - 1.74 \times 10^6 \text{ m} - 6.37 \times 10^6 \text{ m}) = 7.52 \times 10^8 \text{ m}$$

This takes 2.51 s, so $v = \dfrac{7.52 \times 10^8 \text{ m}}{2.51 \text{ s}} = 2.995 \times 10^8 \text{ m/s} = \boxed{299.5 \text{ Mm/s}}$

35.2 $\Delta x = ct$; $c = \dfrac{\Delta x}{t} = \dfrac{2(1.50 \times 10^8 \text{ km})(1000 \text{ m/km})}{(22.0 \text{ min})(60.0 \text{ s/min})} = 2.27 \times 10^8 \text{ m/s} = \boxed{227 \text{ Mm/s}}$

35.3 The experiment is most convincing if the wheel turns fast enough to pass outgoing light through one notch and returning light through the next: $t = 2\ell/c$

$$\theta = \omega t = \omega\left(\frac{2\ell}{c}\right) \quad \text{so} \quad \omega = \frac{c\theta}{2\ell} = \frac{(2.998 \times 10^8)[2\pi/(720)]}{2(11.45 \times 10^3)} = \boxed{114 \text{ rad/s}}$$

The returning light would be blocked by a tooth at one-half the angular speed, giving another data point.

35.4 (a) For the light beam to make it through both slots, the time for the light to travel the distance d must equal the time for the disk to rotate through the angle θ, if c is the speed of light,

$$\frac{d}{c} = \frac{\theta}{\omega}, \quad \text{so} \quad \boxed{c = \frac{d\omega}{\theta}}$$

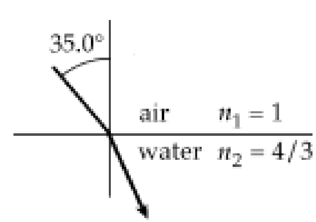
(b) We are given that

$$d = 2.50 \text{ m}, \quad \theta = \frac{1.00°}{60.0}\left(\frac{\pi \text{ rad}}{180°}\right) = 2.91 \times 10^{-4} \text{ rad}, \quad \omega = 5555 \ \frac{\text{rev}}{\text{s}}\left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = 3.49 \times 10^4 \text{ rad/s}$$

$$c = \frac{d\omega}{\theta} = \frac{(2.50 \text{ m})(3.49 \times 10^4 \text{ rad/s})}{2.91 \times 10^{-4} \text{ rad}} = 3.00 \times 10^8 \text{ m/s} = \boxed{300 \text{ Mm/s}}$$

35.5 Using Snell's law, $\sin\theta_2 = \dfrac{n_1}{n_2}\sin\theta_1$

$\theta_2 = \boxed{25.5°}$ $\quad \lambda_2 = \dfrac{\lambda_1}{n_1} = \boxed{442 \text{ nm}}$



35.6 (a) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{6.328 \times 10^{-7} \text{ m}} = \boxed{4.74 \times 10^{14} \text{ Hz}}$

(b) $\lambda_{\text{glass}} = \frac{\lambda_{\text{air}}}{n} = \frac{632.8 \text{ nm}}{1.50} = \boxed{422 \text{ nm}}$

(c) $v_{\text{glass}} = \frac{c_{\text{air}}}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.50} = 2.00 \times 10^8 \text{ m/s} = \boxed{200 \text{ Mm/s}}$

35.7 $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$\sin \theta_1 = 1.333 \sin 45.0°$

$\sin \theta_1 = (1.33)(0.707) = 0.943$

$\theta_1 = 70.5° \rightarrow \boxed{19.5° \text{ above the horizon}}$

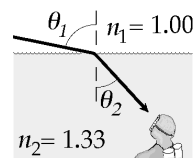


Figure for Goal Solution

Goal Solution

An underwater scuba diver sees the Sun at an apparent angle of 45.0° from the vertical. What is the actual direction of the Sun?

G: The sunlight refracts as it enters the water from the air. Because the water has a higher index of refraction, the light slows down and bends toward the vertical line that is normal to the interface. Therefore, the elevation angle of the Sun above the water will be less than 45° as shown in the diagram to the right, even though it appears to the diver that the sun is 45° above the horizon.

O: We can use Snell's law of refraction to find the precise angle of incidence.

A: Snell's law is: $n_1 \sin \theta_1 = n_2 \sin \theta_2$

which gives $\sin \theta_1 = 1.333 \sin 45.0°$

$\sin \theta_1 = (1.333)(0.707) = 0.943$

The sunlight is at $\theta_1 = 70.5°$ to the vertical, so the Sun is 19.5° above the horizon.

L: The calculated result agrees with our prediction. When applying Snell's law, it is easy to mix up the index values and to confuse angles-with-the-normal and angles-with-the-surface. Making a sketch and a prediction as we did here helps avoid careless mistakes.

***35.8** (a) $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.00 \sin 30.0° = n \sin 19.24°$

$n = \boxed{1.52}$

(c) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{6.328\times10^{-7} \text{ m}} = \boxed{4.74\times10^{14} \text{ Hz}}$ in air and in syrup.

(d) $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.52} = 1.98\times10^8 \text{ m/s} = \boxed{198 \text{ Mm/s}}$

(b) $\lambda = \dfrac{v}{f} = \dfrac{1.98\times10^8 \text{ m/s}}{4.74\times10^{14} \text{/s}} = \boxed{417 \text{ nm}}$

35.9 (a) Flint Glass: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.66} = 1.81\times10^8 \text{ m/s} = \boxed{181 \text{ Mm/s}}$

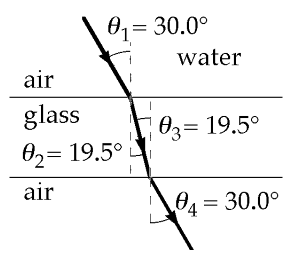
(b) Water: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.333} = 2.25\times10^8 \text{ m/s} = \boxed{225 \text{ Mm/s}}$

(c) Cubic Zirconia: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{2.20} = 1.36\times10^8 \text{ m/s} = \boxed{136 \text{ Mm/s}}$

35.10 $n_1 \sin\theta_1 = n_2 \sin\theta_2$; $1.333 \sin 37.0° = n_2 \sin 25.0°$

$n_2 = 1.90 = \dfrac{c}{v}$; $v = \dfrac{c}{1.90} = 1.58\times10^8 \text{ m/s} = \boxed{158 \text{ Mm/s}}$

35.11 $n_1 \sin\theta_1 = n_2 \sin\theta_2$; $\theta_2 = \sin^{-1}\left(\dfrac{n_1 \sin\theta_1}{n_2}\right)$



$\theta_2 = \sin^{-1}\left\{\dfrac{(1.00)(\sin 30°)}{1.50}\right\} = \boxed{19.5°}$

θ_2 and θ_3 are alternate interior angles formed by the ray cutting parallel normals. So, $\theta_3 = \theta_2 = \boxed{19.5°}$.

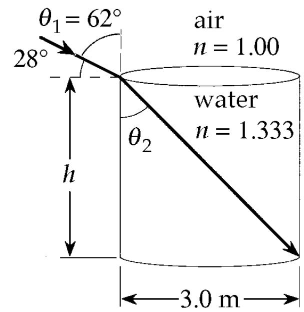
$1.50 \sin\theta_3 = (1.00)\sin\theta_4$

$\theta_4 = \boxed{30.0°}$

35.12 (a) Water $\lambda = \frac{\lambda_0}{n} = \frac{436 \text{ nm}}{1.333} = \boxed{327 \text{ nm}}$

(b) Glass $\lambda = \frac{\lambda_0}{n} = \frac{436 \text{ nm}}{1.52} = \boxed{287 \text{ nm}}$

***35.13** $\sin\theta_1 = n_w \sin\theta_2$



$$\sin\theta_2 = \frac{1}{1.333}\sin\theta_1 = \frac{1}{1.333}\sin(90.0° - 28.0°) = 0.662$$

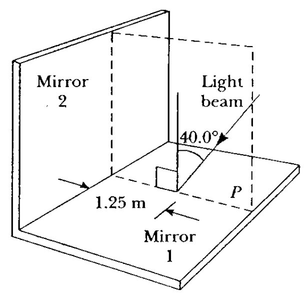
$$\theta_2 = \sin^{-1} 0.662 = 41.5°$$

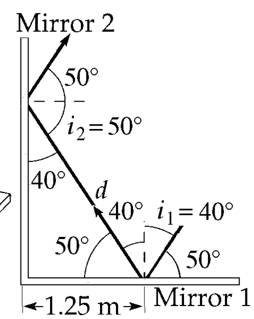
$$h = \frac{d}{\tan\theta_2} = \frac{3.00 \text{ m}}{\tan 41.5°} = \boxed{3.39 \text{ m}}$$

35.14 (a) From geometry, 1.25 m = $d \sin 40.0°$

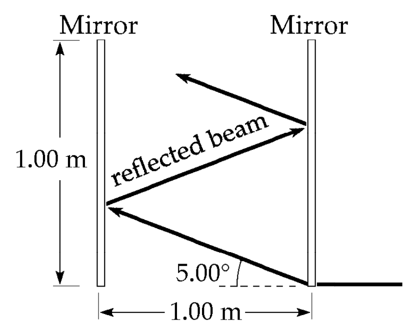
so $d = \boxed{1.94 \text{ m}}$

(b) $\boxed{50.0° \text{ above horizontal}}$, or parallel to the incident ray





***35.15** The incident light reaches the left-hand mirror at distance



$(1.00 \text{ m})\tan 5.00° = 0.0875 \text{ m}$

above its bottom edge. The reflected light first reaches the right-hand mirror at height

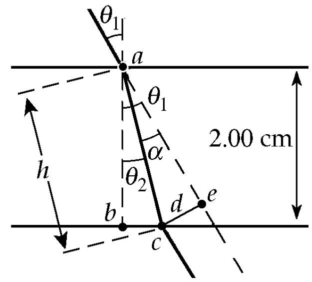
$2(0.0875 \text{ m}) = 0.175 \text{ m}$

It bounces between the mirrors with this distance between points of contact with either.

Since $\frac{1.00 \text{ m}}{0.175 \text{ m}} = 5.72$, the light reflects

$\boxed{\text{five times from the right-hand mirror and six times from the left.}}$

***35.16** At entry, $n_1 \sin\theta_1 = n_2 \sin\theta_2$ or $1.00 \sin 30.0° = 1.50 \sin\theta_2$



$\theta_2 = 19.5°$

The distance h the light travels in the medium is given by

$$\cos\theta_2 = \frac{(2.00\text{ cm})}{h} \quad \text{or} \quad h = \frac{(2.00\text{ cm})}{\cos 19.5°} = 2.12\text{ cm}$$

The angle of deviation upon entry is $\alpha = \theta_1 - \theta_2 = 30.0° - 19.5° = 10.5°$

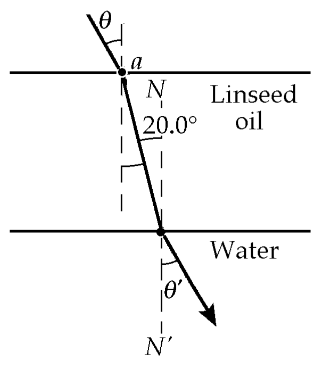
The offset distance comes from $\sin\alpha = \frac{d}{h}$: $d = (2.21\text{ cm})\sin 10.5° = \boxed{0.388\text{ cm}}$

***35.17** The distance, h, traveled by the light is $h = \frac{2.00\text{ cm}}{\cos 19.5°} = 2.12\text{ cm}$

The speed of light in the material is $v = \frac{c}{n} = \frac{3.00\times10^8\text{ m/s}}{1.50} = 2.00\times10^8\text{ m/s}$

Therefore, $t = \frac{h}{v} = \frac{2.12\times10^{-2}\text{ m}}{2.00\times10^8\text{ m/s}} = 1.06\times10^{-10}\text{ s} = \boxed{106\text{ ps}}$

***35.18** Applying Snell's law at the air-oil interface,



$n_{air}\sin\theta = n_{oil}\sin 20.0°$ yields $\boxed{\theta = 30.4°}$

Applying Snell's law at the oil-water interface

$n_w \sin\theta' = n_{oil}\sin 20.0°$ yields $\boxed{\theta' = 22.3°}$

***35.19** time difference = (time for light to travel 6.20 m in ice) – (time to travel 6.20 m in air)

$$\Delta t = \frac{6.20\text{ m}}{v_{ice}} - \frac{6.20\text{ m}}{c} \quad \text{but} \quad v = \frac{c}{n}$$

$$\Delta t = (6.20\text{ m})\left(\frac{1.309}{c} - \frac{1}{c}\right) = \frac{(6.20\text{ m})}{c}(0.309) = 6.39\times10^{-9}\text{ s} = \boxed{6.39\text{ ns}}$$

***35.20** Consider glass with an index of refraction of 1.5, which is 3 mm thick The speed of light in the glass is

$$\frac{3\times10^{8}\ \text{m/s}}{1.5}=2\times10^{8}\ \text{m/s}$$

The extra travel time is $\dfrac{3\times10^{-3}\ \text{m}}{2\times10^{8}\ \text{m/s}}-\dfrac{3\times10^{-3}\ \text{m}}{3\times10^{8}\ \text{m/s}}$ $\boxed{\sim10^{-11}\ \text{s}}$

For light of wavelength 600 nm in vacuum and wavelength $\dfrac{600\ \text{nm}}{1.5}=400\ \text{nm}$ in glass,

the extra optical path, in wavelengths, is $\dfrac{3\times10^{-3}\ \text{m}}{4\times10^{-7}\ \text{m}}-\dfrac{3\times10^{-3}\ \text{m}}{6\times10^{-7}\ \text{m}}$ $\boxed{\sim10^{3}\ \text{wavelengths}}$

***35.21** Refraction proceeds according to $(1.00)\sin\theta_1=(1.66)\sin\theta_2$ (1)

(a) For the normal component of velocity to be constant, $v_1\cos\theta_1=v_2\cos\theta_2$

or $(c)\cos\theta_1=(c/1.66)\cos\theta_2$ (2)

We multiply Equations (1) and (2), obtaining: $\sin\theta_1\cos\theta_1=\sin\theta_2\cos\theta_2$

or $\sin 2\theta_1=\sin 2\theta_2$

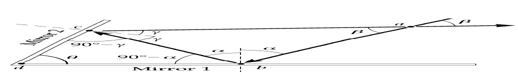
The solution $\theta_1=\theta_2=0$ does not satisfy Equation (2) and must be rejected. The physical solution is $2\theta_1=180°-2\theta_2$ or $\theta_2=90.0°-\theta_1$. Then Equation (1) becomes:

$\sin\theta_1=1.66\cos\theta_1$, or $\tan\theta_1=1.66$

which yields $\theta_1=\boxed{58.9°}$

(b) Light entering the glass slows down and makes a smaller angle with the normal. Both effects reduce the velocity component parallel to the surface of the glass, so that component cannot remain constant, or will remain constant only in the trivial case $\boxed{\theta_1=\theta_2=0}$

35.22 See the sketch showing the path of the light ray. α and γ are angles of incidence at mirrors 1 and 2.



For triangle abca, $2\alpha+2\gamma+\beta=180°$

or $\beta=180°-2(\alpha+\gamma)$ (1)

Now for triangle bcdb,

$$(90.0°-\alpha)+(90.0°-\gamma)+\theta=180°$$

or $\theta=\alpha+\gamma$ (2)

Substituting Equation (2) into Equation (1) gives $\boxed{\beta = 180° - 2\theta}$

Note: From Equation (2), $\gamma = \theta - \alpha$. Thus, the ray will follow a path like that shown only if $\alpha < \theta$. For $\alpha > \theta$, γ is negative and multiple reflections from each mirror will occur before the incident and reflected rays intersect.

35.23 Let $n(x)$ be the index of refraction at distance x below the top of the atmosphere and $n(x = h) = n$ be its value at the planet surface. Then,

$$n(x) = 1.000 + \left(\frac{n - 1.000}{h}\right)x$$

(a) The total time required to traverse the atmosphere is

$$t = \int_0^h \frac{dx}{v} = \int_0^h \frac{n(x)}{c}dx = \frac{1}{c}\int_0^h \left[1.000 + \left(\frac{n - 1.000}{h}\right)x\right]dx = \frac{h}{c} + \frac{(n - 1.000)}{ch}\left(\frac{h^2}{2}\right) = \frac{h}{c}\left(\frac{n + 1.000}{2}\right)$$

$$t = \frac{20.0 \times 10^3 \text{ m}}{3.00 \times 10^8 \text{ m/s}}\left(\frac{1.005 + 1.000}{2}\right) = \boxed{66.8\ \mu\text{s}}$$

(b) The travel time in the absence of an atmosphere would be h/c. Thus, the time in the presence of an atmosphere is

$$\left(\frac{n + 1.000}{2}\right) = 1.0025 \text{ times larger or } \boxed{0.250\% \text{ longer}}\ .$$

35.24 Let $n(x)$ be the index of refraction at distance x below the top of the atmosphere and $n(x = h) = n$ be its value at the planet surface. Then,

$$n(x) = 1.000 + \left(\frac{n - 1.000}{h}\right)x$$

(a) The total time required to traverse the atmosphere is

$$t = \int_0^h \frac{dx}{v} = \int_0^h \frac{n(x)}{c}dx = \frac{1}{c}\int_0^h \left[1.000 + \left(\frac{n - 1.000}{h}\right)x\right]dx = \frac{h}{c} + \frac{(n - 1.000)}{ch}\left(\frac{h^2}{2}\right) = \boxed{\frac{h}{c}\left(\frac{n + 1.000}{2}\right)}$$

(b) The travel time in the absence of an atmosphere would be h/c. Thus, the time in the presence of an atmosphere is

$$\boxed{\left(\frac{n + 1.000}{2}\right) \text{ times larger}}$$

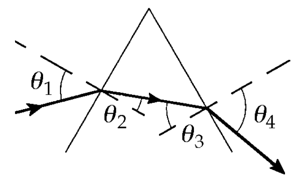
35.25 From Fig. 35.20 $n_v = 1.470$ at 400 nm and $n_r = 1.458$ at 700 nm

Then $(1.00)\sin\theta = 1.470\sin\theta_v$ and $(1.00)\sin\theta = 1.458\sin\theta_r$

$$\delta_r - \delta_v = \theta_r - \theta_v = \sin^{-1}\left(\frac{\sin\theta}{1.458}\right) - \sin^{-1}\left(\frac{\sin\theta}{1.470}\right)$$

$$\Delta\delta = \sin^{-1}\left(\frac{\sin 30.0°}{1.458}\right) - \sin^{-1}\left(\frac{\sin 30.0°}{1.470}\right) = \boxed{0.171°}$$

35.26 $n_1 \sin\theta_1 = n_2 \sin\theta_2$ so $\theta_2 = \sin^{-1}\left(\dfrac{n_1 \sin\theta_1}{n_2}\right)$



$$\theta_2 = \sin^{-1}\left(\frac{(1.00)(\sin 30.0°)}{1.50}\right) = \boxed{19.5°}$$

$$\theta_3 = \left(\left[(90.0° - 19.5°) + 60.0°\right] - 180°\right) + 90.0° = \boxed{40.5°}$$

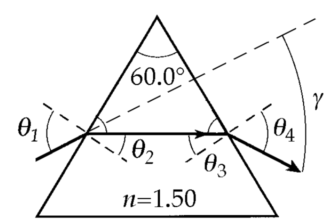
$n_3 \sin\theta_3 = n_4 \sin\theta_4$ so $\theta_4 = \sin^{-1}\left(\dfrac{n_3\sin\theta_3}{n_4}\right) = \sin^{-1}\left(\dfrac{(1.50)(\sin 40.5°)}{1.00}\right) = \boxed{77.1°}$

35.27 Taking Φ to be the apex angle and $\delta_{\min}$ to be the angle of minimum deviation, from Equation 35.9, the index of refraction of the prism material is

$$n = \frac{\sin\left(\frac{\Phi + \delta_{\min}}{2}\right)}{\sin(\Phi/2)}$$

Solving for $\delta_{\min}$, $\delta_{\min} = 2\sin^{-1}\left(n\sin\dfrac{\Phi}{2}\right) - \Phi = 2\sin^{-1}\left[(2.20)\sin(25.0°)\right] - 50.0° = \boxed{86.8°}$

35.28 $n(700 \text{ nm}) = 1.458$



(a) $(1.00)\sin 75.0° = 1.458\sin\theta_2$; $\theta_2 = \boxed{41.5°}$

(b) Let $\theta_3 + \beta = 90.0°$, $\theta_2 + \alpha = 90.0°$; then $\alpha + \beta + 60.0° = 180°$

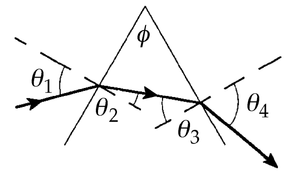
So $60.0° - \theta_2 - \theta_3 = 0 \Rightarrow 60.0° - 41.5° = \theta_3 = \boxed{18.5°}$

(c) $1.458\sin 18.5° = 1.00\sin\theta_4$ $\theta_4 = \boxed{27.6°}$

(d) $\gamma = (\theta_1 - \theta_2) + [\beta - (90.0° - \theta_4)]$

$$\gamma = 75.0° - 41.5° + (90.0° - 18.5°) - (90.0° - 27.6°) = \boxed{42.6°}$$

35.29 For the incoming ray, $\sin\theta_2 = \dfrac{\sin\theta_1}{n}$



Using the figure to the right, $(\theta_2)_{\text{violet}} = \sin^{-1}\left(\dfrac{\sin\ 50.0°}{1.66}\right) = 27.48°$

$$(\theta_2)_{\text{red}} = \sin^{-1}\left(\frac{\sin\ 50.0°}{1.62}\right) = 28.22°$$

For the outgoing ray, $\theta_3' = 60.0° - \theta_2$ and $\sin\theta_4 = n\sin\theta_3$

$$(\theta_4)_{\text{violet}} = \sin^{-1}[1.66\ \sin\ 32.52°] = 63.17°$$

$$(\theta_4)_{\text{red}} = \sin^{-1}[1.62\ \sin\ 31.78°] = 58.56°$$

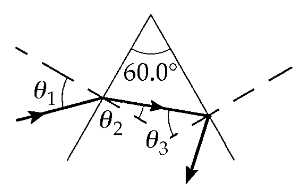
The dispersion is the difference $\Delta\theta_4 = (\theta_4)_{\text{violet}} - (\theta_4)_{\text{red}} = 63.17° - 58.56° = \boxed{4.61°}$

35.30 $n = \dfrac{\sin\left(\dfrac{\Phi + \delta_{\min}}{2}\right)}{\sin(\Phi/2)}$

For small Φ, $\delta_{\min} \approx \Phi$ so $\dfrac{\Phi + \delta_{\min}}{2}$ is also a small angle. Then, using the small angle approximation ($\sin\theta \approx \theta$ when $\theta << 1$ rad), we have:

$n \approx \dfrac{(\Phi + \delta_{\min})/2}{\Phi/2} = \dfrac{\Phi + \delta_{\min}}{\Phi}$ or $\boxed{\delta_{\min} \approx (n-1)\Phi}$ where Φ is in radians.

35.31 At the first refraction, $(1.00)\sin\theta_1 = n\sin\theta_2$



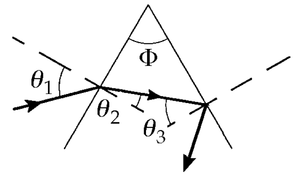
The critical angle at the second surface is given by

$n\sin\theta_3 = 1.00$, or $\theta_3 = \sin^{-1}\left(\dfrac{1.00}{1.50}\right) = 41.8°$.

But, $\theta_2 = 60.0° - \theta_3$. Thus, to avoid total internal reflection at the second surface (i.e., have $\theta_3 < 41.8°$), it is necessary that $\theta_2 > 18.2°$. Since $\sin\theta_1 = n\sin\theta_2$, this requirement becomes

$$\sin\theta_1 > (1.50)\sin(18.2°) = 0.468,\ \text{or}\ \theta_1 > \boxed{27.9°}$$

35.32 At the first refraction, $(1.00)\sin\theta_1 = n\sin\theta_2$. The critical angle at the second surface is given by



$n\sin\theta_3 = 1.00$, or $\theta_3 = \sin^{-1}(1.00/n)$

But $(90.0° - \theta_2) + (90.0° - \theta_3) + \Phi = 180°$, which gives $\theta_2 = \Phi - \theta_3$.

Thus, to have $\theta_3 < \sin^{-1}(1.00/n)$ and avoid total internal reflection at the second surface, it is necessary that $\theta_2 > \Phi - \sin^{-1}(1.00/n)$. Since $\sin\theta_1 = n\sin\theta_2$, this requirement becomes

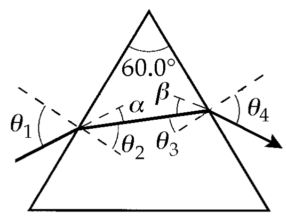
$$\sin\theta_1 > n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right] \quad \text{or} \quad \theta_1 > \boxed{\sin^{-1}\left(n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right]\right)}$$

Through the application of trigonometric identities, $\theta_1 > \boxed{\sin^{-1}\left(\sqrt{n^2-1}\sin\Phi - \cos\Phi\right)}$

35.33 $n = \dfrac{\sin(\delta+\phi)}{\sin(\phi/2)}$ so $1.544\sin\left(\frac{1}{2}\phi\right) = \sin\left(5° + \frac{1}{2}\phi\right) = \cos\left(\frac{1}{2}\phi\right)\sin 5° + \sin\left(\frac{1}{2}\phi\right)\cos 5°$

$\tan\left(\frac{1}{2}\phi\right) = \dfrac{\sin 5°}{1.544 - \cos 5°}$ and $\phi = \boxed{18.1°}$

***35.34** Note for use in every part: $\Phi + (90.0° - \theta_2) + (90.0° - \theta_3) = 180°$



so $\theta_3 = \Phi - \theta_2$

At the first surface is $\alpha = \theta_1 - \theta_2$

At exit, the deviation is $\beta = \theta_4 - \theta_3$

The total deviation is therefore $\delta = \alpha + \beta = \theta_1 + \theta_4 - \theta_2 - \theta_3 = \theta_1 + \theta_4 - \Phi$

(a) At entry: $n_1\sin\theta_1 = n_2\sin\theta_2$ or $\theta_2 = \sin^{-1}\left(\dfrac{\sin 48.6°}{1.50}\right) = 30.0°$

Thus, $\theta_3 = 60.0° - 30.0° = 30.0°$

At exit: $1.50\sin 30.0° = 1.00\sin\theta_4$ or $\theta_4 = \sin^{-1}[1.50\sin(30.0°)] = 48.6°$

so the path through the prism is symmetric when $\theta_1 = 48.6°$.

(b) $\delta = 48.6° + 48.6° - 60.0° = \boxed{37.2°}$

(c) At entry: $\sin\theta_2 = \dfrac{\sin 45.6°}{1.50} \Rightarrow \theta_2 = 28.4°$ $\theta_3 = 60.0° - 28.4° = 31.6°$

At exit: $\sin\theta_4 = 1.50\sin(31.6°) \Rightarrow \theta_4 = 51.7°$ $\delta = 45.6° + 51.7° - 60.0° = \boxed{37.3°}$

(d) At entry: $\sin\theta_2 = \dfrac{\sin 51.6°}{1.50} \Rightarrow \theta_2 = 31.5°$ $\theta_3 = 60.0° - 31.5° = 28.5°$

At exit: $\sin\theta_4 = 1.50\sin(28.5°) \Rightarrow \theta_4 = 45.7°$ $\delta = 51.6° + 45.7° - 60.0° = \boxed{37.3°}$

35.35 $n \sin\theta = 1$. From Table 35.1,

(a) $\theta = \sin^{-1}\left(\dfrac{1}{2.419}\right) = \boxed{24.4°}$

(b) $\theta = \sin^{-1}\left(\dfrac{1}{1.66}\right) = \boxed{37.0°}$

(c) $\theta = \sin^{-1}\left(\dfrac{1}{1.309}\right) = \boxed{49.8°}$

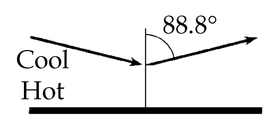
35.36 $\sin\theta_c = \dfrac{n_2}{n_1}$; $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)$

(a) Diamond: $\theta_c = \sin^{-1}\left(\dfrac{1.333}{2.419}\right) = \boxed{33.4°}$

(b) Flint glass: $\theta_c = \sin^{-1}\left(\dfrac{1.333}{1.66}\right) = \boxed{53.4°}$

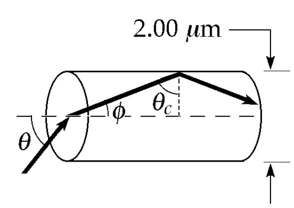
(c) Ice: Since $n_2 > n_1$, $\boxed{\text{there is no critical angle}}$.

35.37 $\sin\theta_c = \dfrac{n_2}{n_1}$ (Equation 35.10)



$n_2 = n_1 \sin 88.8° = (1.0003)(0.9998) = \boxed{1.000\ 08}$

***35.38** $\sin\theta_c = \dfrac{n_{air}}{n_{pipe}} = \dfrac{1.00}{1.36} = 0.735$ $\theta_c = 47.3°$



Geometry shows that the angle of refraction at the end is

$\theta_r = 90.0° - \theta_c = 90.0° - 47.3° = 42.7°$

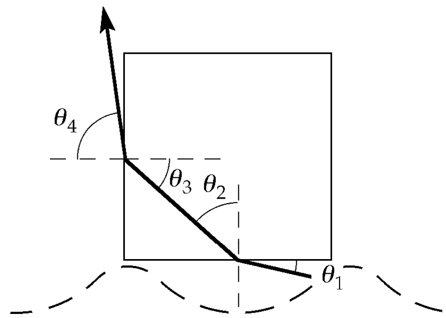
Then, Snell's law at the end, $1.00 \sin\theta = 1.36 \sin 42.7°$

gives $\boxed{\theta = 67.2°}$

35.39 For total internal reflection, $n_1 \sin\theta_1 = n_2 \sin 90.0°$

$(1.50)\sin\theta_1 = (1.33)(1.00)$ or $\theta_1 = \boxed{62.4°}$

35.40 To avoid internal reflection and come out through the vertical face, light inside the cube must have



$\theta_3 < \sin^{-1}(1/n)$

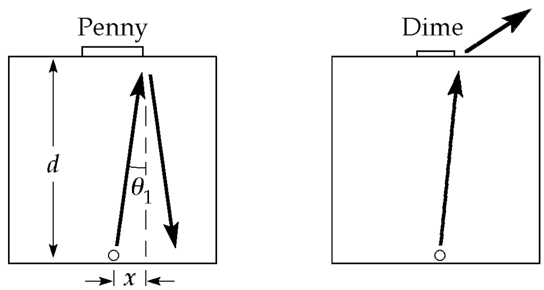
So $\quad \theta_2 > 90.0° - \sin^{-1}(1/n)$

But $\quad \theta_1 < 90.0° \quad$ and $\quad n \sin\theta_2 < 1$

In the critical case, $\sin^{-1}(1/n) = 90.0° - \sin^{-1}(1/n)$

$1/n = \sin 45.0° \qquad \boxed{n = 1.41}$

35.41 From Snell's law, $\quad n_1 \sin\theta_1 = n_2 \sin\theta_2$



At the extreme angle of viewing, $\theta_2 = 90.0°$

$(1.59)(\sin\ \theta_1) = (1.00) \cdot \sin 90.0°$

So $\quad \theta_1 = 39.0°$

Therefore, the depth of the air bubble is

$$\frac{r_d}{\tan\theta_1} < d < \frac{r_p}{\tan\theta_1}$$

or $\quad \boxed{1.08\text{ cm} < d < 1.17\text{ cm}}$

***35.42** (a) $\dfrac{\sin\theta_2}{\sin\theta_1} = \dfrac{v_2}{v_1}$ and $\theta_2 = 90.0°$ at the critical angle

$$\frac{\sin 90.0°}{\sin\theta_c} = \frac{1850\text{ m/s}}{343\text{ m/s}} \quad \text{so} \quad \theta_c = \sin^{-1} 0.185 = \boxed{10.7°}$$

(b) Sound can be totally reflected if it is traveling in the medium where it travels slower: $\boxed{\text{air}}$

(c) $\boxed{\text{Sound in air falling on the wall from most directions is 100\% reflected}}$, so the wall is a good mirror.

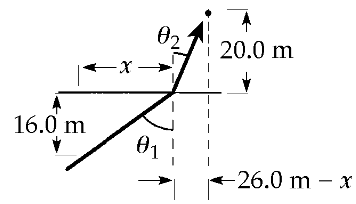
***35.43** For plastic with index of refraction $\boxed{n \geq 1.42}$ surrounded by air, the critical angle for total internal reflection is given by

$$\theta_c = \sin^{-1}\left(\frac{1}{n}\right) \leq \sin^{-1}\left(\frac{1}{1.42}\right) = 44.8°$$

In the gasoline gauge, skylight from above travels down the plastic. The rays close to the vertical are totally reflected from both the sides of the slab and from facets at the lower end of the plastic, where it is not immersed in gasoline. This light returns up inside the plastic and makes it look bright. Where the plastic is immersed in gasoline, with index of refraction about 1.50, total internal reflection should not happen. The light passes out of the lower end of the plastic with little reflected, making this part of the gauge look dark. To frustrate total internal reflection in the gasoline, the index of refraction of the plastic should be $\boxed{n < 2.12}$, since

$$\theta_c = \sin^{-1}\left(\tfrac{1.50}{2.12}\right) = 45.0°.$$

***35.44** Assume the lifeguard's path makes angle θ_1 with the north-south normal to the shoreline, and angle θ_2 with this normal in the water. By Fermat's principle, his path should follow the law of refraction:



$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{7.00\ \text{m/s}}{1.40\ \text{m/s}} = 5.00 \qquad \text{or} \qquad \theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{5}\right)$$

The lifeguard on land travels eastward a distance $x = (16.0\ \text{m})\tan\theta_1$. Then in the water, he travels $26.0\ \text{m} - x = (20.0\ \text{m})\tan\theta_2$ further east. Thus, $26.0\ \text{m} = (16.0\ \text{m})\tan\theta_1 + (20.0\ \text{m})\tan\theta_2$

or $$26.0\ \text{m} = (16.0\ \text{m})\tan\theta_1 + (20.0\ \text{m})\tan\left[\sin^{-1}\left(\frac{\sin\theta_1}{5}\right)\right]$$

We home in on the solution as follows:

θ_1 (deg)	50.0	60.0	54.0	54.8	54.81
right-hand side	22.2 m	31.2 m	25.3 m	25.99 m	26.003 m

The lifeguard should start running at $\boxed{54.8° \text{ east of north}}$.

***35.45** Let the air and glass be medium 1 and 2, respectively. By Snell's law, $n_2 \sin\theta_2 = n_1 \sin\theta_1$

or $1.56 \sin\theta_2 = \sin\theta_1$

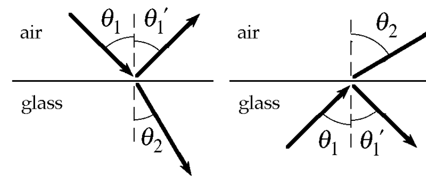
But the conditions of the problem are such that $\theta_1 = 2\theta_2$. $1.56 \sin\theta_2 = \sin 2\theta_2$

We now use the double-angle trig identity suggested. $1.56 \sin\theta_2 = 2\sin\theta_2 \cos\theta_2$

or $$\cos\theta_2 = \frac{1.56}{2} = 0.780$$

Thus, $\theta_2 = 38.7°$ and $\theta_1 = 2\theta_2 = \boxed{77.5°}$

***35.46** (a) $\theta_1' = \theta_1 = \boxed{30.0°}$



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$(1.00)\sin 30.0° = 1.55\ \sin\theta_2$

$\theta_2 = \boxed{18.8°}$

(b) $\theta_1' = \theta_1 = \boxed{30.0°}$

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{1.55 \sin 30.0°}{1}\right) = \boxed{50.8°}$$

(c) and (d) The other entries are computed similarly, and are shown in the table below.

(c) air into glass, angles in degrees			(d) glass into air, angles in degrees		
incidence	reflection	refraction	incidence	reflection	refraction
0	0	0	0	0	0
10.0	10.0	6.43	10.0	10.0	15.6
20.0	20.0	12.7	20.0	20.0	32.0
30.0	30.0	18.8	30.0	30.0	50.8
40.0	40.0	24.5	40.0	40.0	85.1
50.0	50.0	29.6	50.0	50.0	none*
60.0	60.0	34.0	60.0	60.0	none*
70.0	70.0	37.3	70.0	70.0	none*
80.0	80.0	39.4	80.0	80.0	none*
90.0	90.0	40.2	90.0	90.0	none*

*total internal reflection

35.47 For water, $\sin\theta_c = \dfrac{1}{4/3} = \dfrac{3}{4}$

Thus $\theta_c = \sin^{-1}(0.750) = 48.6°$

and $d = 2\left[(1.00\text{ m})\tan\theta_c\right]$

$d = (2.00\text{ m})\tan 48.6° = \boxed{2.27\text{ m}}$

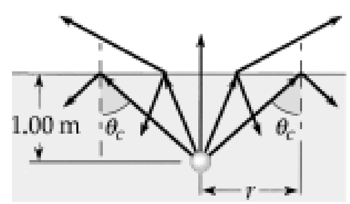


Figure for Goal Solution

Goal Solution

A small underwater pool light is 1.00 m below the surface. The light emerging from the water forms a circle on the water's surface. What is the diameter of this circle?

G: Only the light that is directed upwards and hits the water's surface at less than the critical angle will be transmitted to the air so that someone outside can see it. The light that hits the surface farther from the center at an angle greater than θ_c will be totally reflected within the water, unable to be seen from the outside. From the diagram above, the diameter of this circle of light appears to be about 2 m.

O: We can apply Snell's law to find the critical angle, and the diameter can then be found from the geometry.

A: The critical angle is found when the refracted ray just grazes the surface (θ_2 = 90°). The index of refraction of water is $n_2 = 1.33$, and $n_1 = 1.00$ for air, so

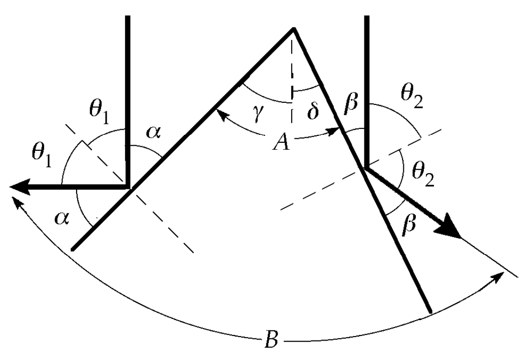
$n_1 \sin\theta_c = n_2 \sin 90°$ gives $\theta_c = \sin^{-1}\left(\frac{1}{1.333}\right) = \sin^{-1}(0.750) = 48.6°$

The radius then satisfies $\tan\theta_c = \frac{r}{(1.00\text{ m})}$

So the diameter is $d = 2r = 2(1.00\text{ m})\tan 48.6° = 2.27\text{ m}$

L: Only the light rays within a 97.2° cone above the lamp escape the water and can be seen by an outside observer (Note: this angle does not depend on the depth of the light source). The path of a light ray is always reversible, so if a person were located beneath the water, they could see the whole hemisphere above the water surface within this cone; this is a good experiment to try the next time you go swimming!

***35.48** Call θ_1 the angle of incidence and of reflection on the left face and θ_2 those angles on the right face. Let α represent the complement of θ_1 and β be the complement of θ_2. Now $\alpha = \gamma$ and $\beta = \delta$ because they are pairs of alternate interior angles. We have



$$A = \gamma + \delta = \alpha + \beta$$

and $B = \alpha + A + \beta = \alpha + \beta + A = \boxed{2A}$

***35.49** (a) We see the Sun swinging around a circle in the extended plane of our parallel of latitude. Its angular speed is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{86\,400 \text{ s}} = 7.27 \times 10^{-5} \text{ rad/s}$$

The direction of sunlight crossing the cell from the window changes at this rate, moving on the opposite wall at speed

$$v = r\omega = (2.37 \text{ m})(7.27 \times 10^{-5} \text{ rad/s}) = 1.72 \times 10^{-4} \text{ m/s} = \boxed{0.172 \text{ mm/s}}$$

(b) The mirror folds into the cell the motion that would occur in a room twice as wide: $v = r\omega = 2(0.174 \text{ mm/s}) = \boxed{0.345 \text{ mm/s}}$

(c) and (d)

As the Sun moves southward and upward at 50.0°, we may regard the corner of the window as fixed, and both patches of light move $\boxed{\text{northward and downward at } 50.0°}$.

***35.50** By Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$

With $v = \dfrac{c}{n}$, $\dfrac{c}{v_1}\sin\theta_1 = \dfrac{c}{v_2}\sin\theta_2$ or $\dfrac{\sin\theta_1}{v_1} = \dfrac{\sin\theta_2}{v_2}$

This is also true for sound. Here, $\dfrac{\sin 12.0°}{340 \text{ m/s}} = \dfrac{\sin\theta_2}{1510 \text{ m/s}}$

$$\theta_2 = \arcsin(4.44 \sin 12.0°) = \boxed{67.4°}$$

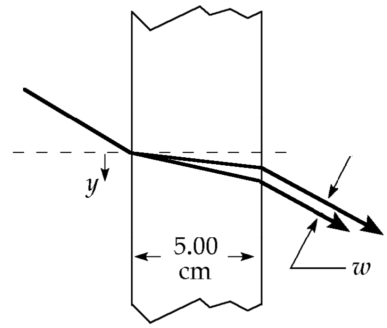
***35.51** (a) $$n = \frac{c}{v} = \frac{2.998 \times 10^8 \text{ m/s}}{\left(61.15 \dfrac{\text{km}}{\text{hr}}\right)\left(\dfrac{1.00 \text{ h}}{3600 \text{ s}}\right)\left(\dfrac{1.00 \times 10^3 \text{ m}}{1.00 \text{ km}}\right)} = \boxed{1.76 \times 10^7}$$

(b) $n_1 \sin\theta_1 = n_2 \sin\theta_2$ so $(1.76 \times 10^7)\sin\theta_1 = (1.00)\sin 90.0°$

$$\theta_1 = \boxed{3.25 \times 10^{-6} \text{ degree}}$$

This problem is misleading. The speed of energy transport is slow, but the speed of the wavefront advance is normally fast. The condensate's index of refraction is not far from unity.

***35.52** Violet light:



$$(1.00)\sin 25.0° = 1.689 \sin \theta_2 \Rightarrow \theta_2 = 14.490°$$

$$y_v = (5.00 \text{ cm})\tan \theta_2 = (5.00 \text{ cm})\tan 14.490° = 1.2622 \text{ cm}$$

Red Light:

$$(1.00)\sin 25.0° = 1.642 \sin \theta_2 \Rightarrow \theta_2 = 14.915°$$

$$y_R = (5.00 \text{ cm})\tan 14.915° = 1.3318 \text{ cm}$$

The emergent beams are both at 25.0° from the normal. Thus,

$w = \Delta y \cos 25.0°$ where $\Delta y = 1.3318 \text{ cm} - 1.2622 \text{ cm} = 0.0396 \text{ cm}$

$$w = (0.396 \text{ mm})\cos 25.0° = \boxed{0.359 \text{ mm}}$$

35.53 Horizontal light rays from the setting Sun pass above the hiker. The light rays are twice refracted and once reflected, as in Figure (b) below, by just the certain special raindrops at 40.0° to 42.0° from the hiker's shadow, and reach the hiker as the rainbow.

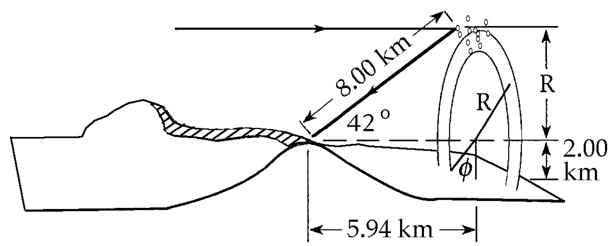


Figure (a)

The hiker sees a greater percentage of the violet inner edge, so we consider the red outer edge. The radius R of the circle of droplets is

$$R = (8.00 \text{ km})(\sin 42.0°) = 5.35 \text{ km}$$

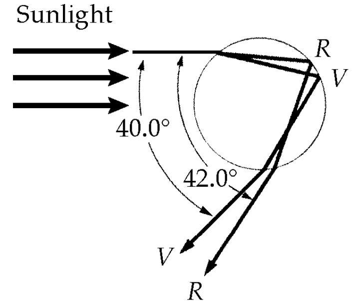


Figure (b)

Then the angle ϕ, between the vertical and the radius where the bow touches the ground, is given by

$$\cos\phi = \frac{2.00 \text{ km}}{R} = \frac{2.00 \text{ km}}{5.35 \text{ km}} = 0.374 \quad \text{or} \quad \phi = 68.1°$$

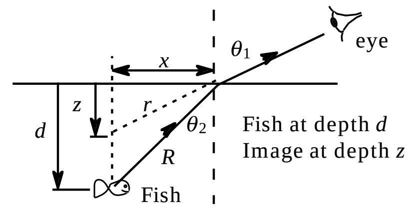
The angle filled by the visible bow is $360° - (2 \times 68.1°) = 224°$, so the visible bow is

$$\frac{224°}{360°} = \boxed{62.2\% \text{ of a circle}}$$

35.54 From Snell's law, $(1.00)\sin\theta_1 = \frac{4}{3}\sin\theta_2$

$x = R\sin\theta_2 = r\sin\theta_1$

so $\frac{r}{R} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{3}{4}$



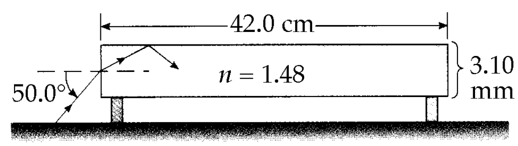
$$\frac{\text{apparent depth}}{\text{actual depth}} = \frac{z}{d} = \frac{r\cos\theta_1}{R\cos\theta_2} = \frac{3}{4}\frac{\cos\theta_1}{\sqrt{1-\sin^2\theta_2}}$$

But $\sin^2\theta_2 = \left(\frac{3}{4}\sin\theta_1\right)^2 = \frac{9}{16}\left(1-\cos^2\theta_1\right)$

So $$\frac{z}{d} = \frac{3}{4}\frac{\cos\theta_1}{\sqrt{1-\frac{9}{16}+\frac{9}{16}\cos^2\theta_1}} = \frac{3}{4}\frac{\cos\theta_1}{\sqrt{\frac{7+9\cos^2\theta_1}{16}}} \quad \text{or} \quad \boxed{z = \frac{3d\cos\theta_1}{\sqrt{7+9\cos^2\theta_1}}}$$

35.55 As the beam enters the slab, $(1.00)\sin 50.0° = (1.48)\sin\theta_2$ giving $\theta_2 = 31.2°$. The beam then strikes the top of the slab at $x_1 = 1.55\text{ mm}/\tan(31.2°)$ from the left end. Thereafter, the beam strikes a face each time it has traveled a distance of $2x_1$ along the length of the slab. Since the slab is 420 mm long, the beam has an additional $420\text{ mm} - x_1$ to travel after the first reflection. The number of additional reflections is

$$\frac{420\text{ mm} - x_1}{2x_1} = \frac{420\text{ mm} - 1.55\text{ mm}/\tan(31.2°)}{3.10\text{ mm}/\tan(31.2°)} = 81.5$$



or 81 reflections since the answer must be an integer. The total number of reflections made in the slab is then $\boxed{82}$.

***35.56** (a) $$\frac{S_1'}{S_1} = \left[\frac{n_2-n_1}{n_2+n_1}\right]^2 = \left[\frac{1.52-1.00}{1.52+1.00}\right]^2 = \boxed{0.0426}$$

(b) If medium 1 is glass and medium 2 is air, $\frac{S_1'}{S_1} = \left[\frac{n_2-n_1}{n_2+n_1}\right]^2 = \left[\frac{1.00-1.52}{1.00+1.52}\right]^2 = 0.0426;$

There is $\boxed{\text{no difference}}$

(c) $$\frac{S_1'}{S_1} = \left[\frac{1.76\times10^7-1.00}{1.76\times10^7+1.00}\right]^2 = \left[\frac{1.76\times10^7+1.00-2.00}{1.76\times10^7+1.00}\right]^2$$

$$\frac{S_1'}{S_1} = \left[1.00 - \frac{2.00}{1.76\times10^7+1.00}\right]^2 \approx 1.00 - 2\left(\frac{2.00}{1.76\times10^7+1.00}\right) = 1.00 - 2.27\times10^{-7} \quad \text{or} \quad \boxed{100\%}$$

This suggests he appearance would be $\boxed{\text{very shiny, reflecting practically all incident light}}$. See, however, the note concluding the solution to problem 35.51.

***35.57** (a) With $n_1 = 1$ and $n_2 = n$, the reflected fractional intensity is $\frac{S_1'}{S_1} = \left(\frac{n-1}{n+1}\right)^2$.

The remaining intensity must be transmitted:

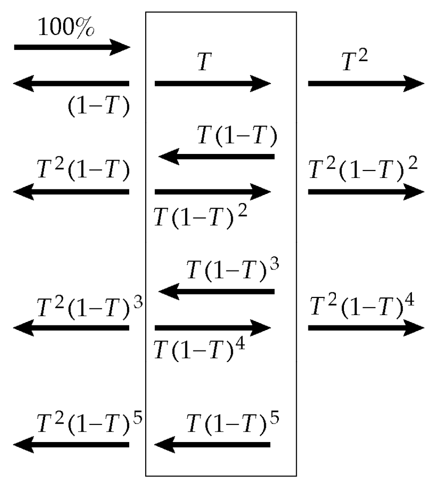
$$\frac{S_2}{S_1} = 1 - \left(\frac{n-1}{n+1}\right)^2 = \frac{(n+1)^2 - (n-1)^2}{(n+1)^2} = \frac{n^2 + 2n + 1 - n^2 + 2n - 1}{(n+1)^2} = \boxed{\frac{4n}{(n+1)^2}}$$

(b) At entry, $\frac{S_2}{S_1} = 1 - \left(\frac{n-1}{n+1}\right)^2 = \frac{4(2.419)}{(2.419+1)^2} = 0.828$

At exit, $\frac{S_3}{S_2} = 0.828$

Overall, $\frac{S_3}{S_1} = \left(\frac{S_3}{S_2}\right)\left(\frac{S_2}{S_1}\right) = (0.828)^2 = 0.685$ or $\boxed{68.5\%}$

***35.58** Define $T = \frac{4n}{(n+1)^2}$ as the transmission coefficient for one encounter with an interface. For diamond and air, it is 0.828, as in problem 57.



As shown in the figure, the total amount transmitted is

$$T^2 + T^2(1-T)^2 + T^2(1-T)^4 + T^2(1-T)^6 + \ldots + T^2(1-T)^{2n} + \ldots$$

We have $1 - T = 1 - 0.828 = 0.172$ so the total transmission is

$$(0.828)^2\left[1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots\right]$$

To sum this series, define $F = 1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots$.
Note that $(0.172)^2 F = (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots$, and
$1 + (0.172)^2 F = 1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots = F$.

Then, $1 = F - (0.172)^2 F$ or $F = \frac{1}{1-(0.172)^2}$

The overall transmission is then $\frac{(0.828)^2}{1-(0.172)^2} = 0.706$ or $\boxed{70.6\%}$

35.59 $n \sin 42.0° = \sin 90.0°$ so $n = \dfrac{1}{\sin 42.0°} = 1.49$

$\sin\theta_1 = n \sin 18.0°$ and $\sin\theta_1 = \dfrac{\sin 18.0°}{\sin 42.0°}$

$\theta_1 = \boxed{27.5°}$

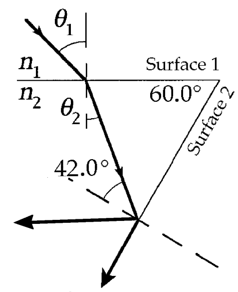


Figure for Goal Solution

<u>Goal Solution</u>

The light beam shown in Figure P35.59 strikes surface 2 at the critical angle. Determine the angle of incidence θ_1.

G: From the diagram it appears that the angle of incidence is about 40°.

O: We can find θ_1 by applying Snell's law at the first interface where the light is refracted. At surface 2, knowing that the 42.0° angle of reflection is the critical angle, we can work backwards to find θ_1.

A: Define n_1 to be the index of refraction of the surrounding medium and n_2 to be that for the prism material. We can use the critical angle of 42.0° to find the ratio n_2/n_1:

$$n_2 \sin 42.0° = n_1 \sin 90.0°$$

So, $$\frac{n_2}{n_1} = \frac{1}{\sin 42.0°} = 1.49$$

Call the angle of refraction θ_2 at the surface 1. The ray inside the prism forms a triangle with surfaces 1 and 2, so the sum of the interior angles of this triangle must be 180°. Thus,

$$(90.0° - \theta_2) + 60.0° + (90.0° - 42.0°) = 180°$$

Therefore, $\theta_2 = 18.0°$

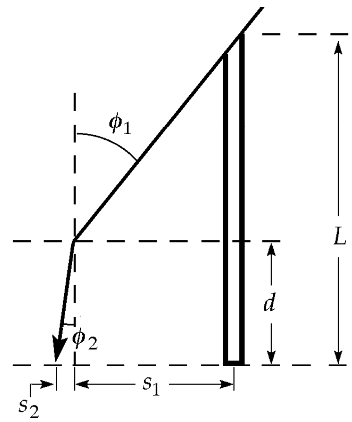
Applying Snell's law at surface 1, $n_1 \sin\theta_1 = n_2 \sin 18.0°$

$$\sin\theta_1 = (n_2/n_1)\sin\theta_2 = (1.49)\sin 18.0°$$

$$\theta_1 = 27.5°$$

L: The result is a bit less than the 40.0° we expected, but this is probably because the figure is not drawn to scale. This problem was a bit tricky because it required four key concepts (refraction, reflection, critical angle, and geometry) in order to find the solution. One practical extension of this problem is to consider what would happen to the exiting light if the angle of incidence were varied slightly. Would all the light still be reflected off surface 2, or would some light be refracted and pass through this second surface?

35.60 Light passing the top of the pole makes an angle of incidence $\phi_1 = 90.0° - \theta$. It falls on the water surface at distance



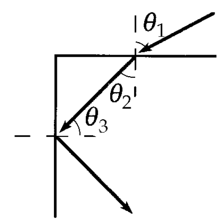
$$s_1 = \frac{(L-d)}{\tan\theta}$$ from the pole,

and has an angle of refraction ϕ_2 from $(1.00)\sin\phi_1 = n\sin\phi_2$. Then $s_2 = d\tan\phi_2$ and the whole shadow length is

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\sin\phi_1}{n}\right)\right)$$

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\cos\theta}{n}\right)\right) = \frac{2.00\text{ m}}{\tan 40.0°} + (2.00\text{ m})\tan\left(\sin^{-1}\left(\frac{\cos 40.0°}{1.33}\right)\right) = \boxed{3.79\text{ m}}$$

35.61 (a) For polystyrene *surrounded by air*, internal reflection requires



$$\theta_3 = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2°$$

Then from the geometry, $\theta_2 = 90.0° - \theta_3 = 47.8°$

From Snell's law, $\sin\theta_1 = (1.49)\sin 47.8° = 1.10$

This has no solution. Therefore, total internal reflection $\boxed{\text{always happens}}$.

(b) For polystyrene *surrounded by water*, $\theta_3 = \sin^{-1}\left(\frac{1.33}{1.49}\right) = 63.2°$

and $\theta_2 = 26.8°$

From Snell's law, $\theta_1 = \boxed{30.3°}$

(c) $\boxed{\text{No internal refraction is possible}}$ since the beam is initially traveling in a medium of lower index of refraction.

***35.62** $\delta = \theta_1 - \theta_2 = 10.0°$ and $n_1\sin\theta_1 = n_2\sin\theta_2$ with $n_1 = 1$, $n_2 = \frac{4}{3}$

Thus, $\theta_1 = \sin^{-1}(n_2\sin\theta_2) = \sin^{-1}[n_2\sin(\theta_1 - 10.0°)]$

(You can use a calculator to home in on an approximate solution to this equation, testing different values of θ_1 until you find that $\theta_1 = \boxed{36.5°}$. Alternatively, you can solve for θ_1 exactly, as shown below.)

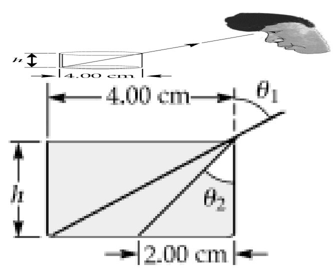
We are given that $\sin\theta_1 = \frac{4}{3}\sin(\theta_1 - 10.0°)$

This is the sine of a difference, so $\frac{3}{4}\sin\theta_1 = \sin\theta_1 \cos 10.0° - \cos\theta_1 \sin 10.0°$

Rearranging, $\sin 10.0° \cos\theta_1 = \left(\cos 10.0° - \frac{3}{4}\right)\sin\theta_1$

$$\frac{\sin 10.0°}{\cos 10.0° - 0.750} = \tan\theta_1 \quad \text{and} \quad \theta_1 = \tan^{-1} 0.740 = \boxed{36.5°}$$

35.63 $\tan\theta_1 = \frac{4.00\text{ cm}}{h}$ and $\tan\theta_2 = \frac{2.00\text{ cm}}{h}$



$$\tan^2\theta_1 = (2.00\tan\theta_2)^2 = 4.00\tan^2\theta_2$$

$$\frac{\sin^2\theta_1}{(1 - \sin^2\theta_1)} = 4.00\frac{\sin^2\theta_2}{(1 - \sin^2\theta_2)} \qquad (1)$$

Snell's law in this case is: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\sin\theta_1 = 1.333 \sin\theta_2$$

Squaring both sides, $\sin^2\theta_1 = 1.777 \sin^2\theta_2$ (2)

Substituting (2) into (1), $\frac{1.777\sin^2\theta_2}{1 - 1.777\sin^2\theta_2} = 4.00\frac{\sin^2\theta_2}{1 - \sin^2\theta_2}$

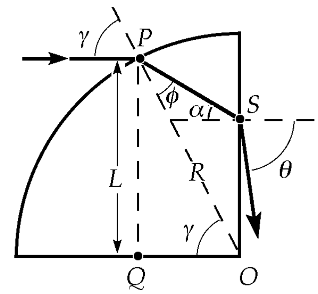
Defining $x = \sin^2\theta$, $\frac{0.444}{(1 - 1.777x)} = \frac{1}{(1 - x)}$

Solving for x, $0.444 - 0.444x = 1 - 1.777x$ and $x = 0.417$

From x we can solve for θ_2: $\theta_2 = \sin^{-1}\sqrt{0.417} = 40.2°$

Thus, the height is $h = \frac{(2.00\text{ cm})}{\tan\theta_2} = \frac{(2.00\text{ cm})}{\tan(40.2°)} = \boxed{2.37\text{ cm}}$

35.64 Observe in the sketch that the angle of incidence at point P is γ, and using triangle OPQ: $\sin\gamma = L/R$.



Also, $\cos\gamma = \sqrt{1-\sin^2\gamma} = \dfrac{\sqrt{R^2-L^2}}{R}$

Applying Snell's law at point P, $(1.00)\sin\gamma = n\sin\phi$

Thus, $\sin\phi = \dfrac{\sin\gamma}{n} = \dfrac{L}{nR}$

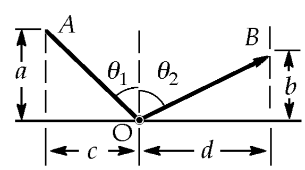
and $\cos\phi = \sqrt{1-\sin^2\phi} = \dfrac{\sqrt{n^2R^2-L^2}}{nR}$

From triangle OPS, $\phi + (\alpha + 90.0°) + (90.0° - \gamma) = 180°$ or the angle of incidence at point S is $\alpha = \gamma - \phi$. Then, applying Snell's law at point S gives $(1.00)\sin\theta = n\sin\alpha = n\sin(\gamma-\phi)$, or

$$\sin\theta = n[\sin\gamma\cos\phi - \cos\gamma\sin\phi] = n\left[\left(\frac{L}{R}\right)\frac{\sqrt{n^2R^2-L^2}}{nR} - \frac{\sqrt{R^2-L^2}}{R}\left(\frac{L}{nR}\right)\right]$$

$$\sin\theta = \frac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right) \quad \text{and} \quad \boxed{\theta = \sin^{-1}\left[\frac{L}{R^2}\left(\sqrt{n^2R^2-L^2}-\sqrt{R^2-L^2}\right)\right]}$$

35.65 To derive the law of *reflection*, locate point O so that the time of travel from point A to point B will be minimum.



The *total* light path is $L = a\sec\theta_1 + b\sec\theta_2$

The time of travel is $t = \left(\dfrac{1}{v}\right)(a\sec\theta_1 + b\sec\theta_2)$

If point O is displaced by dx, then

$$dt = \left(\frac{1}{v}\right)(a\sec\theta_1\tan\theta_1\,d\theta_1 + b\sec\theta_2\tan\theta_2\,d\theta_2) = 0 \qquad (1)$$

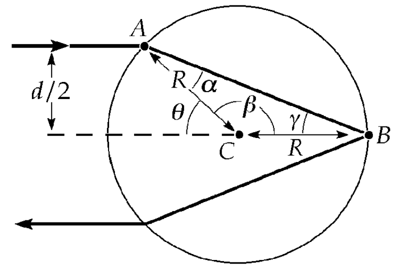
(since for minimum time $dt = 0$).

Also, $c + d = a\tan\theta_1 + b\tan\theta_2 = \text{constant}$

so, $$a\sec^2\theta_1\,d\theta_1 + b\sec^2\theta_2\,d\theta_2 = 0 \qquad (2)$$

Divide equations (1) and (2) to find $\boxed{\theta_1 = \theta_2}$

35.66 As shown in the sketch, the angle of incidence at point A is:



$$\theta = \sin^{-1}\left[\frac{(d/2)}{R}\right] = \sin^{-1}\left[\frac{1.00\text{ m}}{2.00\text{ m}}\right] = 30.0°$$

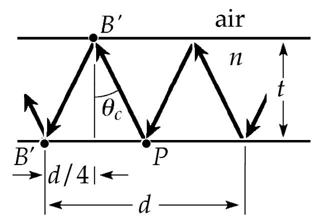
If the emerging ray is to be parallel to the incident ray, the path must be symmetric about the center line CB of the cylinder . In the isosceles triangle ABC, $\gamma = \alpha$ and $\beta = 180° - \theta$. Therefore, $\alpha + \beta + \gamma = 180°$ becomes

$$2\alpha + 180° - \theta = 180° \quad \text{or} \quad \alpha = \frac{\theta}{2} = 15.0°$$

Then, applying Snell's law at point A, $n \sin\alpha = (1.00)\sin\theta$

or $$n = \frac{\sin\theta}{\sin\alpha} = \frac{\sin 30.0°}{\sin 15.0°} = \boxed{1.93}$$

35.67 (a) At the boundary of the air and glass, the critical angle is given by



$$\sin\theta_c = \frac{1}{n}$$

Consider the critical ray PBB': $\tan\theta_c = \frac{d/4}{t}$ or $\frac{\sin\theta_c}{\cos\theta_c} = \frac{d}{4t}$

Squaring the last equation gives: $\frac{\sin^2\theta_c}{\cos^2\theta_c} = \frac{\sin^2\theta_c}{1-\sin^2\theta_c} = \left(\frac{d}{4t}\right)^2$

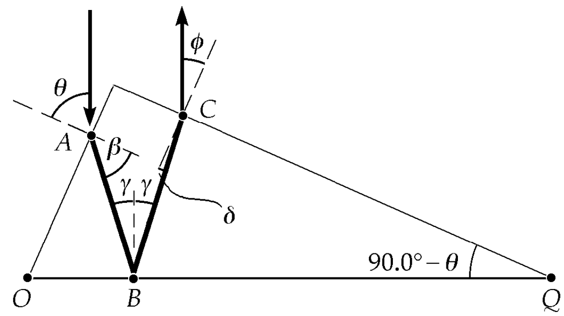
Since $\sin\theta_c = \frac{1}{n}$, this becomes $\frac{1}{n^2-1} = \left(\frac{d}{4t}\right)^2$ or $\boxed{n = \sqrt{1+(4t/d)^2}}$

(b) Solving for d, $d = \frac{4t}{\sqrt{n^2-1}}$

Thus, if $n = 1.52$ and $t = 0.600$ cm, $d = \frac{4(0.600\text{ cm})}{\sqrt{(1.52)^2-1}} = \boxed{2.10\text{ cm}}$

(c) Since violet light has a larger index of refraction, it will lead to a smaller critical angle and the inner edge of the white halo will be tinged with $\boxed{\text{violet}}$ light.

35.68 From the sketch, observe that the angle of incidence at point A is the same as the prism angle θ at point O. Given that $\theta = 60.0°$, application of Snell's law at point A gives



$1.50 \sin \beta = 1.00 \sin 60.0°$ or $\beta = 35.3°$

From triangle AOB, we calculate the angle of incidence (and reflection) at point B.

$\theta + (90.0° - \beta) + (90.0° - \gamma) = 180°$ so $\gamma = \theta - \beta = 60.0° - 35.3° = 24.7°$

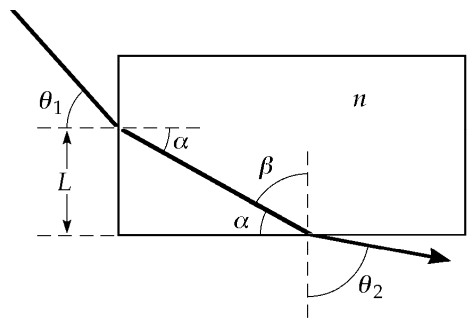
Now, using triangle BCQ: $(90.0° - \gamma) + (90.0° - \delta) + (90.0° - \theta) = 180°$

Thus the angle of incidence at point C is $\delta = (90.0° - \theta) - \gamma = 30.0° - 24.7° = 5.30°$

Finally, Snell's law applied at point C gives $1.00 \sin \phi = 1.50 \sin 5.30°$

or $\phi = \sin^{-1}(1.50 \sin 5.30°) = \boxed{7.96°}$

35.69 (a) Given that $\theta_1 = 45.0°$ and $\theta_2 = 76.0°$, Snell's law at the first surface gives



$$n \sin \alpha = (1.00) \sin 45.0° \quad (1)$$

Observe that the angle of incidence at the second surface is $\beta = 90.0° - \alpha$. Thus, Snell's law at the second surface yields

$$n \sin \beta = n \sin(90.0° - \alpha) = (1.00) \sin 76.0°, \text{ or}$$

$$n \cos \alpha = \sin 76.0° \quad (2)$$

Dividing Equation (1) by Equation (2), $\tan \alpha = \dfrac{\sin 45.0°}{\sin 76.0°} = 0.729$ or $\alpha = 36.1°$

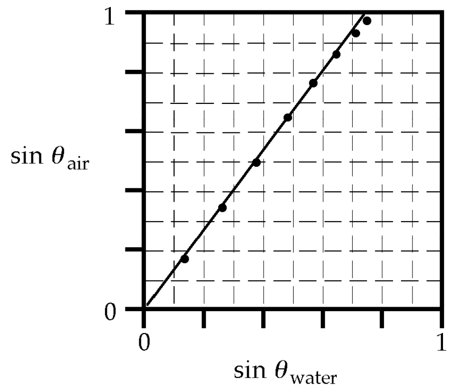
Then, from Equation (1), $n = \dfrac{\sin 45.0°}{\sin \alpha} = \dfrac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$

(b) From the sketch, observe that the distance the light travels in the plastic is $d = L/\sin \alpha$. Also, the speed of light in the plastic is $v = c/n$, so the time required to travel through the plastic is

$$t = \frac{d}{v} = \frac{nL}{c \sin \alpha} = \frac{(1.20)(0.500 \text{ m})}{(3.00 \times 10^8 \text{ m/s}) \sin 36.1°} = 3.40 \times 10^{-9} \text{ s} = \boxed{3.40 \text{ ns}}$$

35.70

$\sin\theta_1$	$\sin\theta_2$	$\sin\theta_1 / \sin\theta_2$
0.174	0.131	1.3304
0.342	0.261	1.3129
0.500	0.379	1.3177
0.643	0.480	1.3385
0.766	0.576	1.3289
0.866	0.647	1.3390
0.940	0.711	1.3220
0.985	0.740	1.3315



The straightness of the graph line demonstrates Snell's proportionality. The slope of the line is $\bar{n} = 1.3276 \pm 0.01$

and $n = \boxed{1.328 \pm 0.8\%}$